Escaping the Large Fine-Tuning and Little Hierarchy Problems in the Next to Minimal Supersymmetric Model and $h \to aa$ Decays

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We demonstrate that the NMSSM can have small fine-tuning and modest light stop mass while still evading all experimental constraints. For small $\tan\beta$ (large $\tan\beta$), the relevant scenarios are such that there is always (often) a SM-like Higgs boson that decays to two lighter — possibly much lighter — pseudoscalar Higgses.

In the CP-conserving Minimal Supersymmetric Model (MSSM), large soft-supersymmetry-breaking mass parameters are required in order that the one-loop corrections to the tree-level prediction for the lightest Higgs boson ($m_h \leq m_Z$) increase m_h sufficiently to avoid conflict with lower bounds from LEP data. The large size of these soft-SUSY breaking masses compared to the weak scale, the natural scale where supersymmetry is expected, is termed the little-hierarchy problem. This hierarchy implies that a substantial amount of fine-tuning of the MSSM soft-SUSY breaking parameters is needed. The severity of these problems has led to a variety of alternative approaches. For instance, little Higgs models [1] can be less fine tuned. Or, one can argue that large fine-tuning is not so bad, as in "split-supersymmetry" [2]. In this letter, we show that the Next to Minimal Supersymmetric Model (NMSSM [3]) can avoid or at least ameliorate the fine-tuning and little hierarchy problems. In addition, we find that parameter choices that are consistent with all LEP constraints and that yield small fine-tuning at small $\tan\beta$ (large $\tan\beta$) are nearly always (often) such that there is a relatively light SM-like CP-even Higgs boson that decays into two light, perhaps very light, pseudoscalars. Such decays dramatically complicate the Tevatron and LHC searches for Higgs bosons.

The NMSSM is very attractive in its own right. It provides a very elegant solution to the μ problem of the MSSM via the introduction of a singlet superfield $\widehat{S}$. For the simplest possible scale invariant form of the superpotential, the scalar component of $\widehat{S}$ naturally acquires a vacuum expectation value of the order of the SUSY breaking scale, giving rise to a value of μ of order the electroweak scale. The NMSSM is the simplest supersymmetric extension of the standard model in which the electroweak scale originates from the SUSY breaking scale only. A possible cosmological domain wall problem [4] can be avoided by introducing suitable non-renormalizable operators [5] that do not generate dangerously large singlet tadpole diagrams [6]. Hence, the phenomenology of the NMSSM deserves to be studied at least as fully and precisely as that of the MSSM.

Radiative corrections to the Higgs masses have been computed [7, 8, 9, 10] and basic phenomenology of the model has been studied [11]. The NMHDECAY program [12] allows easy exploration of Higgs phenomenology in the NMSSM. In particular, it allows for the possibility of Higgs to Higgs pair decay modes (first emphasized in [13] and studied later in [14]) and includes the associated modifications of LEP limits. Of greatest relevance are $h \to aa$ decays, where h is a SM-like CP-even Higgs boson and a is a (mostly singlet) CP-odd Higgs boson. The relevant limits come from the analysis [15] of the $Zh \to Zaa \to Zb\overline{b}b\overline{b}$ channel and the analysis [16] of the $Zh \to Zaa \to Z\tau^+\tau^-\tau^+\tau^-$ channel. The weaker nature of the limits from LEP on such scenarios will play an important role in what follows.

The extent to which there is a no-lose theorem for NMSSM Higgs discovery at the LHC has arisen as an important topic [13, 17, 18, 19, 20]. In particular, it has been found that the Higgs to Higgs pair decay modes can render inadequate the usual MSSM Higgs search modes that give rise to a no-lose theorem for MSSM Higgs discovery at the LHC. And, it is by no means proven that the Higgs to Higgs pair modes are directly observable at the LHC, although there is some hope [18, 19].

Earlier discussions of fine-tuning in the NMSSM have been given in [21, 22].

We very briefly review the NMSSM. Its particle content differs from the MSSM by the addition of one CP-even and one CP-odd state in the neutral Higgs sector (assuming CP conservation), and one additional neutralino. We will follow the conventions of [12]. Apart from the usual quark and lepton Yukawa couplings, the scale invariant superpotential is

$$\lambda\ \widehat{S}\widehat{H}_u\widehat{H}_d + \frac{\kappa}{3}\ \widehat{S}^3 \tag{1}$$

depending on two dimensionless couplings λ, κ beyond the MSSM. [Hatted (unhatted) capital letters denote superfields (scalar superfield components).] The associated trilinear soft terms are

$$\lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3\,. \tag{2}$$

The final two input parameters are

$$\tan\beta = h_u/h_d\,, \quad \mu_{\rm eff} = \lambda s\,, \tag{3}$$

where $h_u \equiv \langle H_u\rangle$, $h_d \equiv \langle H_d\rangle$ and $s \equiv \langle S\rangle$. These, along with m_Z, can be viewed as determining the three SUSY breaking masses squared for H_u, H_d and S (denoted $m^2_{H_u}$, $m^2_{H_d}$ and m^2_S) through the three minimization equations of the scalar potential.

Thus, as compared to the three independent parameters needed in the MSSM context (often chosen as μ,

$\tan\beta$ and M_A), the Higgs sector of the NMSSM is described by the six parameters

$$\lambda\ ,\ \kappa\ ,\ A_\lambda\ ,\ A_\kappa,\ \tan\beta\ ,\ \mu_{\rm eff}\ . \tag{4}$$

We will choose sign conventions for the fields such that λ and $\tan\beta$ are positive, while κ, A_λ, A_κ and $\mu_{\rm eff}$ should be allowed to have either sign. In addition, values must be input for the gaugino masses and for the soft terms related to the (third generation) squarks and sleptons that contribute to the radiative corrections in the Higgs sector and to the Higgs decay widths.

Sample discussions of the fine-tuning issues for the MSSM appear in [23]. We will define

$$F = {\rm Max}_a F_a \equiv {\rm Max}_a \left|\frac{d\log m_Z}{d\log a}\right| , \tag{5}$$

where the parameters a comprise μ, B_μ and the other GUT-scale soft-SUSY-breaking parameters. (In some papers, $\frac{d\log m_Z^2}{d\log a}$ is employed.) In our approach, we choose m_Z-scale values for all the squark soft masses squared, the gaugino masses, $M_{1,2,3}(m_Z)$, $A_t(m_Z)$ and $A_b(m_Z)$ (with no requirement of universality at the GUT scale). We also choose m_Z-scale values for $\tan\beta$, μ and m_A; these uniquely determine $B_\mu(m_Z)$. The vevs h_u and h_d at scale m_Z are fixed by $\tan\beta$ and m_Z via $m_Z^2=\overline{g}^2(h_u^2+h_d^2)$ (where $\overline{g}^2=g^2+g'^2$). Finally, $m_{H_u}^2(m_Z)$ and $m_{H_d}^2(m_Z)$ are determined by the two potential minimization conditions. We then evolve all parameters to the MSSM GUT scale (including μ and B_μ). Next, we shift each of the GUT-scale parameters in turn, evolve back down to scale m_Z, and reminimize the Higgs potential using the shifted values of μ, B_μ, $m_{H_u}^2$ and $m_{H_d}^2$. This gives new values for h_u and h_d yielding new values for m_Z and $\tan\beta$.

Results will be presented for $\tan\beta(m_Z)=10$, $M_{1,2,3}(m_Z)=100,200,300$ GeV. We scan randomly over $|A_t(m_Z)|\leq 500$ GeV and 3rd generation squark and slepton soft masses-squared above $(200\ {\rm GeV})^2$, as well as over $|\mu(m_Z)|\geq 100$ GeV, sign$(\mu)=\pm$ and over $m_A>120$ GeV (for which LEP, MSSM constraints require $m_h\gtrsim 114$ GeV [24]). On the left side of Fig. 1, we plot F as a function of the mean stop mass $\sqrt{m_{\widetilde t_1}m_{\widetilde t_2}}$, which enters into the computation (we use HDECAY [25] with $m_t^{\rm pole}=175$ GeV) of the radiative correction to the SM-like light Higgs mass m_h. Points plotted as $+$'s ($\times$'s) have $m_h<114$ GeV ($m_h\geq 114$ GeV) and are excluded (allowed) by LEP data. Very modest values of F (of order $F\sim 5$) are possible for $m_h<114$ GeV but the smallest F value found for $m_h\geq 114$ GeV is of order $F\sim 185$ [27]. The very rapid increase of the smallest achievable F with m_h is illustrated in the right plot of Fig. 1. This is the essence of the current fine-tuning problem for the CP-conserving MSSM. Also, to achieve $m_h>114$ GeV, $\sqrt{m_{\widetilde t_1}m_{\widetilde t_2}}\gtrsim 1$ TeV is required, an indicator of the little hierarchy problem.

We now contrast this to the NMSSM situation. One combination of the three potential minimization equations yields the usual MSSM-like expression for m_Z^2 in terms of μ^2, $\tan\beta$, $m_{H_u}^2$ and $m_{H_d}^2$, with μ replaced by $\mu_{\rm eff}$. However, a second combination gives an expression for $\mu_{\rm eff}$ in terms of m_Z^2 and other Higgs potential parameters. Eliminating $\mu_{\rm eff}$, we arrive at an equation of the form $m_Z^4+2Bm_Z^2+C=0$, with solution $m_Z^2=-B\pm\sqrt{B^2-C}$, where B and C are given in terms of the soft susy breaking parameters, λ, κ and $\tan\beta$. Only one of the solutions to the quadratic equation applies for any given set of parameter choices. Small fine-tuning is typically achieved when $C\ll B^2$ and derivatives of m_Z^2 with respect to a GUT scale parameter tend to cancel between the $-B$ and $+\sqrt{B^2-C}$ ($-\sqrt{B^2-C}$) for $B>0$ (for $B<0$).

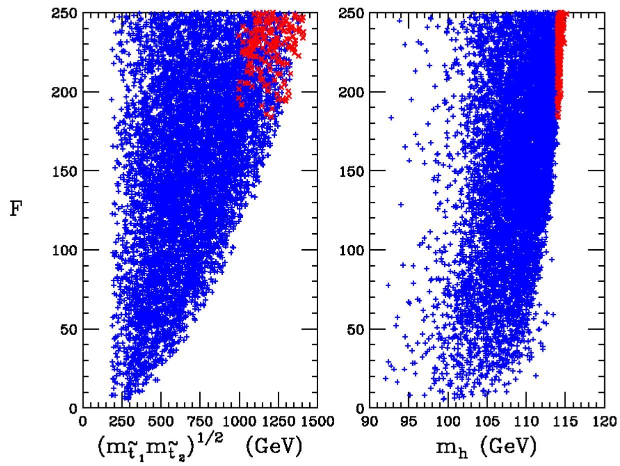

FIG. 1: Left: the fine-tuning measure F in the MSSM is plotted vs. $\sqrt{m_{\widetilde t_1}m_{\widetilde t_2}}$, without regard to LEP constraints on m_h. The $+$ points have $m_h<114$ GeV and are excluded by LEP limits. The $\times$ points have $m_h>114$ GeV and are experimentally allowed. Right: F is plotted vs. m_h for all scanned points.

To explore fine-tuning, we proceed analogously to the manner described for the MSSM. At scale m_Z, we fix $\tan\beta$ and scan over values of $\lambda\leq 0.5$ ($\lambda\lesssim 0.7$ is required for perturbativity up to the GUT scale), $|\kappa|\leq 0.3$, sign$(\kappa)=\pm$ and $100\ {\rm GeV}\leq|\mu_{\rm eff}|\leq 1.5$ TeV, sign$(\mu_{\rm eff})=\pm$. We also choose m_Z-scale values for the soft-SUSY-breaking parameters A_λ, A_κ, $A_t=A_b$, M_1, M_2, M_3, m_Q^2, m_U^2, m_D^2, m_L^2, and m_E^2, all of which enter into the evolution equations. We process each such choice through NMHDECAY (using $m_t^{\rm pole}=175$ GeV) to check that the scenario satisfies all theoretical and available experimental constraints (including $m_{\widetilde t_1}\geq 100$ GeV). For accepted cases, we then evolve to determine the GUT-scale values of all the above parameters. The fine-tuning derivative for each parameter is determined by shifting the GUT-scale value for that parameter by a small amount, evolving all parameters back down to m_Z, redetermining the potential minimum (which gives new values h_u' and h_d') and finally computing a new value for m_Z^2 using $m_Z'^2=\overline{g}^2(h_u'^2+h_d'^2)$.

Our results for $\tan\beta=10$ and $M_{1,2,3}(m_Z)=100,200,300$ GeV and randomly chosen values for the soft-SUSY-breaking parameters listed earlier are dis-

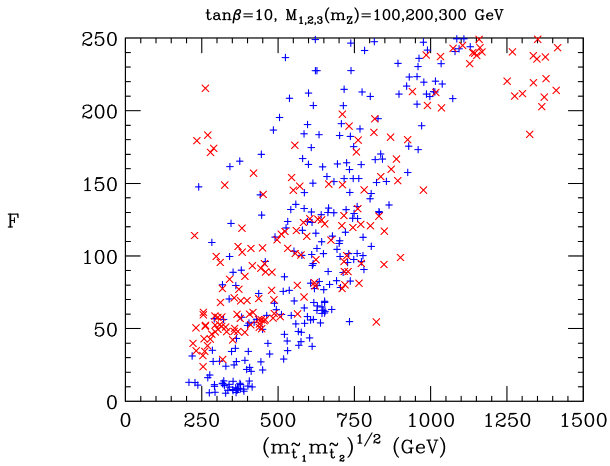

FIG. 2: For the NMSSM, we plot the fine-tuning measure F vs. $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}$ for NMHDECAY-accepted scenarios with $\tan\beta = 10$ and $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV. Points marked by '+' ('×') escape LEP exclusion primarily due to dominance of $h_1 \to a_1 a_1$ decays (due to $m_{h_1} > 114$ GeV).

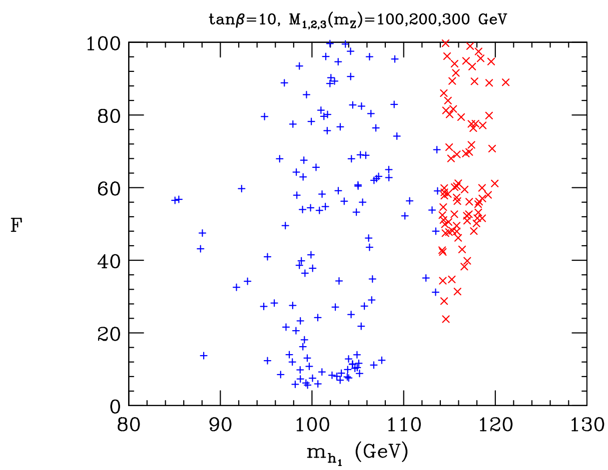

FIG. 3: For the NMSSM, we plot the fine-tuning measure F vs. m_{h_1} for NMHDECAY-accepted scenarios with $\tan\beta = 10$ and $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV. Point labeling as in Fig. 2.

played in Fig. 2. We see that F as small as $F \sim 5.5$ can be achieved for $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}} \sim 250 \div 400$ GeV. In the figure, the + points have $m_{h_1} < 114$ GeV and escape LEP exclusion by virtue of the dominance of $h_1 \to a_1 a_1$ decays; as noted earlier, LEP is less sensitive to this channel as compared to the traditional $h_1 \to b\overline{b}$ decays. Points marked by × have $m_{h_1} > 114$ GeV and will escape LEP exclusion regardless of the dominant decay mode. For most of these latter points $h_1 \to b\overline{b}$ decays are dominant, even if somewhat suppressed; $h_1 \to a_1 a_1$ decays dominate for a few. For both classes of points, the h_1 has fairly SM-like couplings. We also note that all points with $F < 20$ have $m_{h_1} < 114$ GeV and $BR(h_1 \to a_1 a_1) > 0.70$. Finally, in Fig. 3 we demonstrate the rapid increase of the minimum F with m_{h_1}. The lowest F values are only achieved for $m_{h_1} \lesssim 105$ GeV. However, even for $m_{h_1} \geq 114$ GeV, the lowest F value of $F \sim 24$ is far below that attainable for $m_h \geq 114$ GeV in the MSSM.

A small value for $A_\kappa(m_Z)$ (typically of order a few GeV) appears to be essential to achieve small F. First, small A_κ allows small enough m_{a_1} [28] that $h_1 \to a_1 a_1$ decays are dominant; this makes it possible for the naturally less fine-tuned values of $m_{h_1} < 114$ GeV to be LEP-allowed. Second, small F is frequently (nearly always) achieved for $m_{h_1} < 114$ GeV ($m_{h_1} \geq 114$ GeV) via the cancellation mechanism noted earlier, where $C \ll B^2$, and this mechanism generally works mainly for small A_κ. Indeed, there are many phenomenologically acceptable parameter choices with $m_{h_1} > 114$ GeV that have large A_κ, but these all also have very large F.

For lower $\tan\beta$ values such as $\tan\beta = 3$, extremely large $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}$ is required for $m_h > 114$ GeV in the MSSM, leading to extremely large F. Results in the NMSSM for $\tan\beta = 3$ are plotted in Fig. 4 for $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV and scanning as in the $\tan\beta = 10$ case. We see that $F \sim 15$ is achievable for $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}} \sim 300$ GeV. No points with $m_{h_1} > 114$ GeV were found. All the plotted points escape LEP limits because of the dominance of the $h_1 \to a_1 a_1$ decay. For very large $\tan\beta$ (*e.g.* $\tan\beta \sim 50$), it is possible to obtain $m_h > 114$ GeV with relatively small $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}$ in the MSSM as well as in the NMSSM. We have not yet studied fine-tuning at very large $\tan\beta$ in either model.

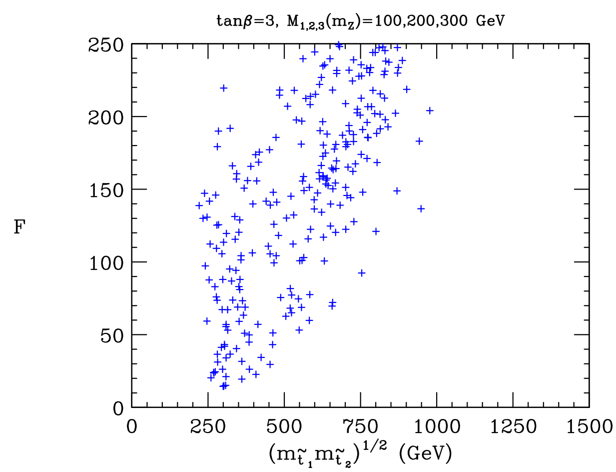

FIG. 4: For the NMSSM, we plot the fine-tuning measure F vs. $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}$ for NMHDECAY-accepted scenarios with $\tan\beta = 3$ and $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV. Point labeling as in Fig. 2.

In the NMSSM context, the smallest achievable value for F is mainly sensitive to $M_3(m_Z)$. For example, for $M_3(m_Z) \sim 700$ GeV and $\tan\beta = 10$, the smallest F we find is of order $F \sim 40$.

We note that in [21] the mass of the SM-like Higgs h (where $h = h_2$ for the parameter choices they focus on) is increased beyond the LEP limit by choosing modest $\tan\beta \sim 2 \div 5$ and λ values close to the 0.7 upper limit consistent with perturbativity up to the GUT scale. This maximizes the additional NMSSM tree-level

contribution to m_h^2 proportional to λ^2, thereby allowing $m_h > 114$ GeV for somewhat smaller $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}$ than in the MSSM. This, in turn, reduces the fine-tuning and little hierarchy problems, but not nearly to the extent achieved by our parameter choices. In our plots, the SM-like h is always the h_1. The points with very small F have low $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}$, modest λ and κ, and escape LEP constraints not because m_h is large but because $h \to aa$ decays are dominant.

In conclusion, we reemphasize that the NMSSM provides a rather simple escape from the large fine-tuning and (little) hierarchy problems characteristic of the CP-conserving MSSM. However, the relevant NMSSM models imply a high probability for $h_1 \to a_1 a_1$ decays to be dominant. We speculate that similar results will emerge in many supersymmetric models where the Higgs sector is more complicated than that of the MSSM. Higgs detection in such a decay mode should be pursued with greatly increased vigor. Existing work [18, 19] which suggests a very marginal LHC signal for $WW \to h_1 \to a_1 a_1 \to b\overline{b}\tau^+\tau^-$ when $m_{a_1} > 2m_b$ should be either refuted or improved upon. In addition, the $a_1 a_1 \to \tau^+\tau^-\tau^+\tau^-$ channel that dominates for $2m_\tau < m_{a_1} < 2m_b$ (an entirely acceptable and rather frequently occurring mass range in our parameter scans and not excluded by Υ decays since the a_1 has a large singlet component) should receive immediate attention. Hopefully, we will not have to wait for Higgs discovery at an e^+e^- linear collider via the inclusive $Zh \to \ell^+\ell^- X$ reconstructed M_X approach (which allows Higgs discovery independent of the Higgs decay mode) or at a CLIC-based $\gamma\gamma$ collider [26] in the $\gamma\gamma \to h \to b\overline{b}\tau^+\tau^-$ or $\tau^+\tau^-\tau^+\tau^-$ modes.

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